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Sun's Motion and Sunspots*

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The investigation discloses that the variation in the motion of the sun about the center of mass of the solar system has a periodicity of 178.7 yr. The sunspot cycle is found to have the same period. Although the oscillations in the sunspot numbers and in the sun's motion are not in perfect agreement, the few departures that do occur, occur under very similar conditions.

I. INTRODUCTION

In the determination of planetary positions the sun and planets are considered to be point masses. No consideration is taken of their diameters and the nature of their masses, whether solid, gaseous, or liquid. In the lunar theory, size and shape do enter into the solution but no account is taken of the liquid and gaseous parts which are not rigidly attached to the main mass. The actual motion of the sun is quite complicated as noted by Newton (see Cajori 1934): ". . . since that centre of gravity (center of mass of the solar system) is continually at rest, the sun, according to the various positions of the planets, must continually move every way, but will never recede far from that centre."

In the next section the path of the sun projected on the invariable plane is represented as suggested in the quotation from Newton's *Principia*. In Sec. III several parameters of the sun's motion are computed and it is shown that these have a period of 178.7 yr. In Sec. IV the relative sunspot numbers are examined in relation to the rate of change of the sun's instantaneous angular momentum. It is found that these two quantities have very nearly the same period.

The author (Jose 1936) first looked into this problem in 1936, but satisfactory results were not achieved then because of the lack of data presented in a uniform manner over an extended period of time and the large amount of computing required.

II. SUN'S MOTION

The publication of "Coordinates of the Five Outer Planets 1653–2060" by Eckert, Brouwer, and Clemence (1951), and the availability of high-speed computers, have made possible a more thorough study of the sun's motion than was possible in 1936. The "Coordinates of the Five Outer Planets" tabulates the heliocentric equatorial rectangular coordinates of the planets at 40-day intervals. These coordinates, given in astronomical units, are referred to the mean equinox and equator of 1950.0. With these data the coordinates of the sun relative to the center of mass of the solar system were computed. In order that the path of the sun could be represented graphically in two dimensions with minimum distortion, the coordinate axes were rotated so that the xy plane coincided with the invariable plane and the x axis coincided with the ascending node of the invariable plane on the ecliptic. Clemence and Brouwer (1955) give the elements of the invariable plane as: $I=1^\circ 39' 16''.47$, $\Omega=107^\circ 16' 38''.96$. The obliquity of the ecliptic is, $\epsilon=23^\circ 26' 44''.84$.

TABLE I. Julian day number and calendar date for points on the curves of Fig. 1.

Point	Fig. 1(a)	Fig. 1(b)	Fig. 1(c)	Fig. 1(d)
1	239 1200.5 1834 Oct. 13.5	240 7200.5 1878 Aug. 3.5	242 4800.5 1926 Oct. 12.0	244 0800.5 1970 Aug. 2.0
2	239 2800.5 1839 Mar. 1.5	240 8800.5 1882 Dec. 20.5	242 6400.5 1931 Feb. 28.0	244 2400.5 1974 Dec. 19.0
3	239 4400.5 1843 Jul. 18.5	241 0400.5 1887 May 8.5	242 8000.5 1935 Jul. 17.0	244 4000.5 1979 May 7.0
4	239 6000.5 1847 Dec. 4.5	241 2000.5 1891 Sep. 24.5	242 9600.5 1939 Dec. 3.0	244 5600.5 1983 Sep. 23.0
5	239 7600.5 1852 Apr. 21.5	241 3600.5 1896 Feb. 10.5	243 1200.5 1944 Apr. 20.0	244 7200.5 1988 Feb. 9.0
6	239 9200.5 1856 Sep. 7.5	241 5200.5 1900 Jun. 29.5	243 2800.5 1948 Sep. 6.0	244 8800.5 1992 Jun. 27.0
7	240 0800.5 1861 Jan. 24.5	241 6800.5 1904 Nov. 15.5	243 4400.5 1953 Jan. 23.0	245 0400.5 1996 Nov. 13.0
8	240 2400.5 1865 Jun. 12.5	241 8400.5 1909 Apr. 3.5	243 6000.5 1957 Jun. 11.0	245 2000.5 2001 Apr. 1.0
9	240 4000.5 1869 Oct. 29.5	242 000.5 1913 Aug. 20.5	243 7600.5 1961 Oct. 28.0	245 3600.5 2005 Aug. 18.0
10	240 5600.5 1874 Mar. 17.5	242 1600.5 1918 Jan. 6.5	243 9200.5 1966 Mar. 16.0	245 5200.5 2010 Jan. 4.0
11	240 7200.5 1878 Aug. 3.5	242 3200.5 1922 May 25.5	244 0000.5 1968 May 24.0	245 6400.5 2013 Apr. 18.0

In Fig. 1 the path of the sun with respect to the center of mass of the solar system is represented for 178.7 yr (1833–2013). This interval, as will be shown, is the period of variation of the sun's motion about the center of mass of the solar system. Table I gives the dates corresponding to the numbered positions of Fig. 1. The points in this figure are plotted at 200-day intervals. Corresponding curves for the preceding 178.7-yr period would be nearly identical to those shown except for a rotation of 30° counterclockwise which is due primarily to the synodic period of Uranus and Neptune. In Fig. 1(d) it is interesting to note that in 1990 the sun will have a retrograde motion relative

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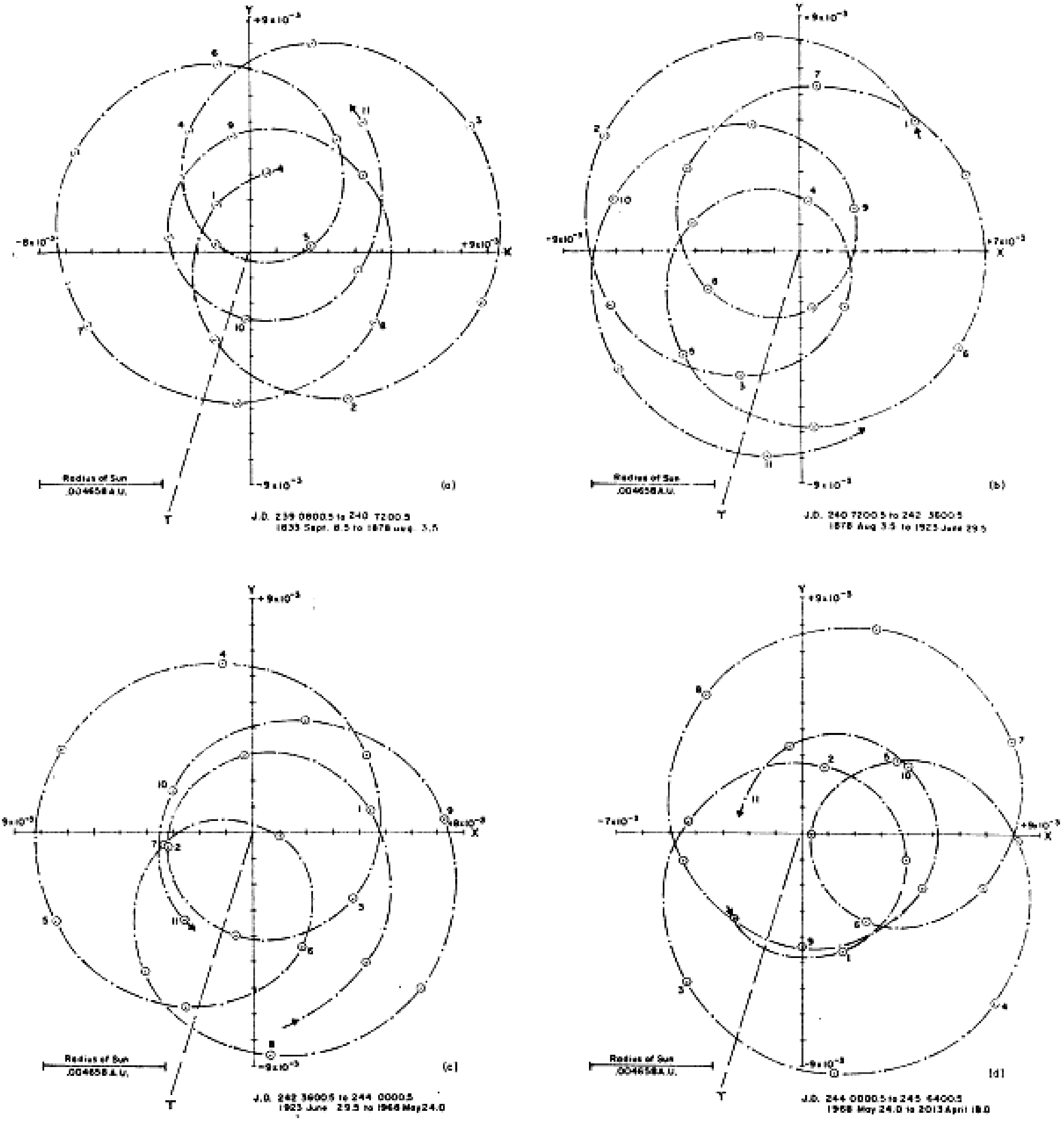

J.D. 239 0800.5 to 240 7200.5
1833 Sept. 8.5 to 1878 Aug. 3.5

J.D. 240 7200.5 to 242 3600.5
1878 Aug 3.5 to 1923 June 29.5

J.D. 242 3600.5 to 244 0000.5
1923 June 29.5 to 1968 May 24.0

J.D. 244 0000.5 to 245 6400.5
1968 May 24.0 to 2013 April 18.0

FIG. 1. Path of the sun's center about the center of mass of the solar system. Numbers along the curve refer to the dates listed in Table I. At the lower left of each drawing is a line which represents the sun's radius. The direction of the vernal equinox, projected on the invariable plane, is indicated. The unit of length is the astronomical unit.

to the center of mass; that is, its angular momentum referred to the center of mass will be negative. The previous similar situation occurred in 1811.

The following characteristics of the sun's motion are noted. The distance between the center of the sun and the center of mass varies from 0.01 to 2.19 solar radii. The inclination of the osculating plane of the sun's motion to the invariable plane varies between $0^\circ\!.15$ and $0^\circ\!.75$; and the ascending node of the osculating plane oscillates through an angle of 80°.

III. ANGULAR MOMENTUM OF THE SUN

The total angular momentum of the solar system is constant. However, the angular momentum of an individual member of the system referred to the center of mass is variable. With the coordinates (x,y,z) of the sun relative to the center of mass being known, for equally spaced intervals of time, any function descriptive of the sun's motion may be found. The following functions have been considered:

R, the distance from the center of mass to the center

Table II. Period determined from four functions.

Function	Mean period (yr)	Standard deviation	Number of intervals
R	178.81	0.32	27
ρ	178.77	0.19	22
dL/dt	178.76	0.43	31
dP/dt	178.70	0.30	23
Combined	178.77	0.34	103

of the sun

$$R=(x^2+y^2+z^2)^{\frac{1}{2}};$$

V the velocity of the sun

$$V=(\dot{x}^2+\dot{y}^2+\dot{z}^2)^{\frac{1}{2}};$$

ρ the radius of curvature of the sun's path

$$\rho=V^3/\Delta,\quad \Delta=[(\dot{y}\ddot{z}-\dot{z}\ddot{y})^2+(\dot{z}\ddot{x}-\dot{x}\ddot{z})^2+(\dot{x}\ddot{y}-\dot{y}\ddot{x})^2]^{\frac{1}{2}};$$

L the angular momentum of the sun about the center of mass

$$L=[(y\dot{z}-z\dot{y})^2+(z\dot{x}-x\dot{z})^2+(x\dot{y}-y\dot{x})^2]^{\frac{1}{2}};$$

dL/dt, the rate of change of L;
P the angular momentum about the instantaneous center of curvature

$$P=\rho V;$$

dP/dt, the rate of change of P.

The unit of mass is the sun's mass. The unit of length is the astronomical unit. The unit of time is 40 days.

After plotting the various functions it was observed that there was a period of approximately 178 yr in the data. The data cover the time from 1653 to 2060 so that there are approximately two and one quarter 178-yr periods. The time intervals between corresponding points of maxima and minima in the successive periods gave an average length of the period of 178.77 yr. Table II summarizes the results.

It was observed that this period of 178.77 yr is nine times the synodic period of Jupiter and Saturn ($9\times 19.858=178.72$). It should be noted that the synodic period of Uranus and Neptune is 171.40 yr.

The functions R, ρ, dL/dt, and dP/dt for the two periods 1655 to 1833 and 1833 to 2012 have been plotted separately in Figs. 2(a) and 2(b), respectively. The time of beginning of the first period was arbitrarily chosen as JD 232 5600 (5 March 1655). A comparison of Figs. 2(a) and 2(b) emphasizes the striking similarity of the computed quantities for the two periods.

Figure 3 represents the three Z-like configurations of the four major planets, Jupiter, Saturn, Uranus, and Neptune at corresponding epochs in three successive 178.7-yr periods. Since this period for the sun's motion (or the equivalent nine synodic periods of Jupiter and Saturn) is 7.3 yr longer than the 171.4-yr synodic period of Uranus and Neptune, it is observed in Fig. 3 that the position of Uranus and Neptune advance relative to that of Jupiter and Saturn (a gradual compression of the Z configuration) and also the distance between Uranus and Neptune increases with the advent of successive epochs. These facts will doubtless cause variations in the curves shown in Figs. 2(a) and 2(b) after an interval of time which is long compared to the 178.7-yr period.

IV. RELATIVE SUNSPOT NUMBERS

In 1852, Wolf at Zurich collected all available observations on sunspot activity back to the year 1610, the time of the first observations. From these data he assigned times of maximum and minimum activity from 1610 to 1749, and after 1749 when the information became richer, he assigned relative sunspot numbers as

Table III. Maxima and minima of sunspot activity adapted from Waldmeier (1941).

Number of cycle	Year of minima	Year of maxima	Differences between corresponding dates of minima	Differences between corresponding dates of maxima
1	1610.8	1615.5		
2	1619.0	1626.0		
3	1634.0	1639.5		
4	1645.0	1649.0		
5	1655.0	1660.0		
6	1666.0	1675.0		
7	1679.5	1685.0		
8	1689.5	1693.0		
9	1698.0	1705.5		
10	1712.0	1718.2		
11	1723.5	1727.5		
12	1734.0	1738.7		
13	1745.0	1750.3		
14	1755.2	1761.5		
15	1766.5	1769.7		
16	1775.5	1778.4		
17 (1)[a]	1784.7	1788.1	173.9[b]	172.6[b]
18 (2)	1798.3	1805.2	179.3	179.2
19 (3)	1810.6	1816.4	176.6[b]	176.9[b]
20 (4)	1823.3	1829.9	178.3	180.9
21 (5)	1833.9	1837.2	178.9	177.2
22 (6)	1843.5	1848.1	177.5	173.1[b]
23 (7)	1856.0	1860.1	176.5	175.1[b]
24 (8)	1867.2	1870.6	177.7	177.6
25 (9)	1878.9	1883.9	180.9	178.4
26 (10)	1889.6	1894.1	177.6	175.9[b]
27 (11)	1901.7	1907.0	178.2	179.5
28 (12)	1913.6	1917.6	179.6	178.9
29 (13)	1923.6	1928.4	178.6	178.1
30 (14)	1933.8	1937.4	178.6	175.9[b]
31 (15)	1944.3	1947.7	177.8	178.0
32 (16)	1954.3	1957.5	178.8	179.1
			Average of 24	
	Predicted		intervals	
33 (1)	1963	1967		$=178.55$
34 (2)	1977	1984		$\sigma=$ 1.05
35 (3)	1990	1995		
36 (4)	2002	2009		

[a] Corresponding cycle in first period.
[b] Dates omitted from average.

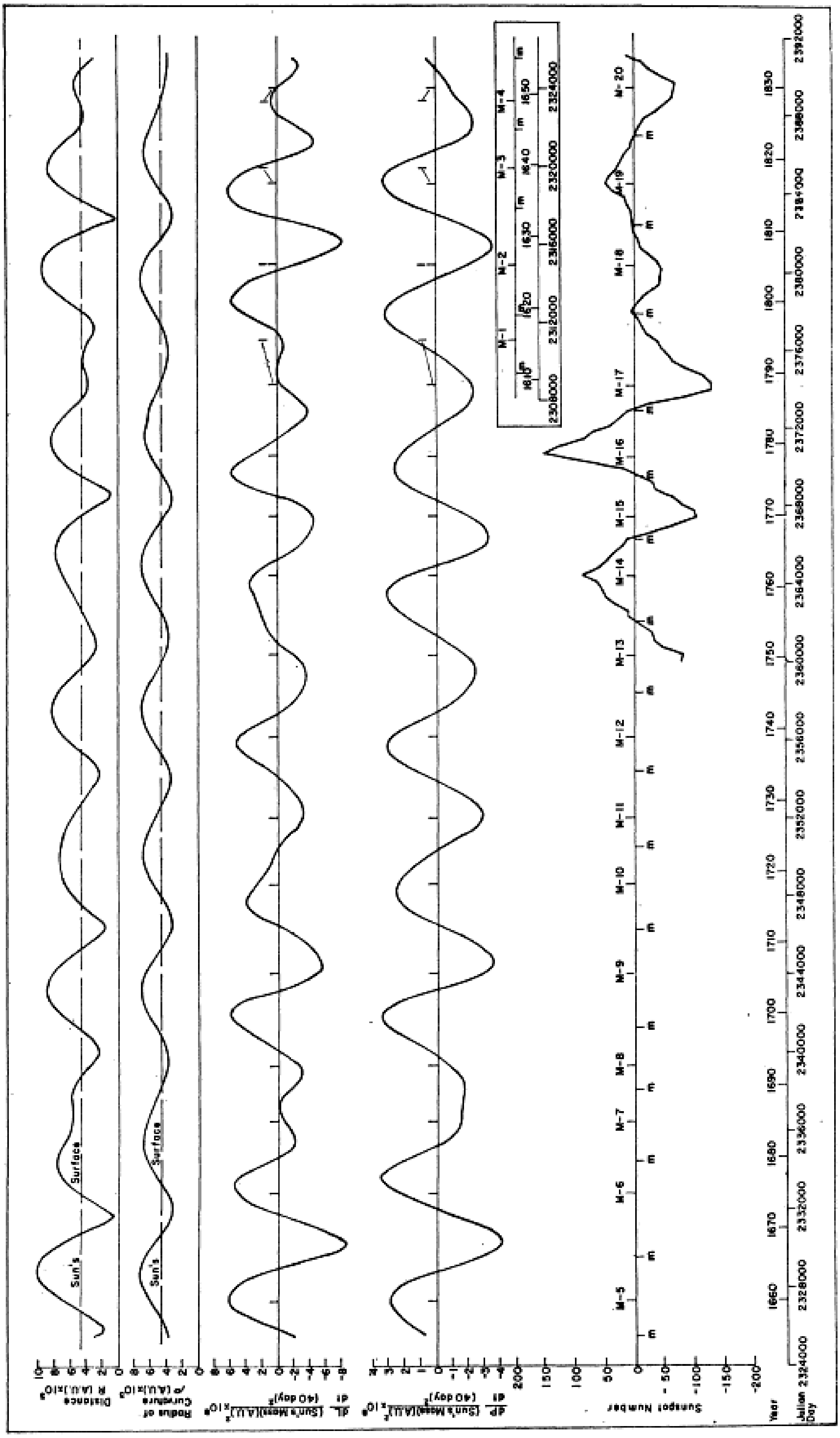

FIG. 2(a). The quantities R, ρ, dL/dt, and dP/dt for the period 1655 to 1833. Relative sunspot numbers and their times of maxima and minima are indicated. The insert shows times of maxima and minima prior to 1655 and is placed in position to agree with the 178.7-yr period.

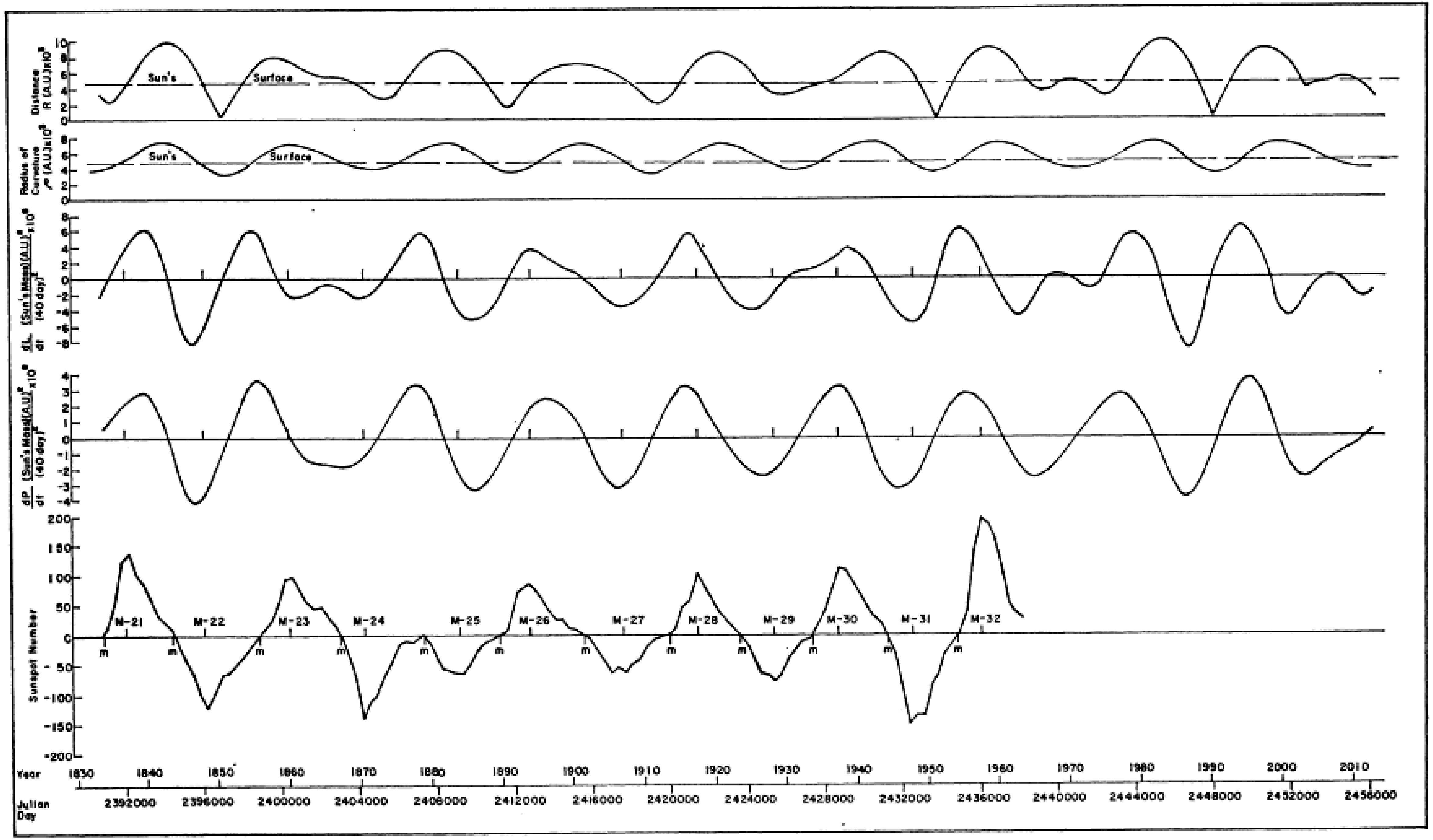

FIG. 2(b). The quantities R, ρ, dL/dt, and dP/dt for the period 1833 to 2012. Relative sunspot numbers and times of maxima and minima are indicated.

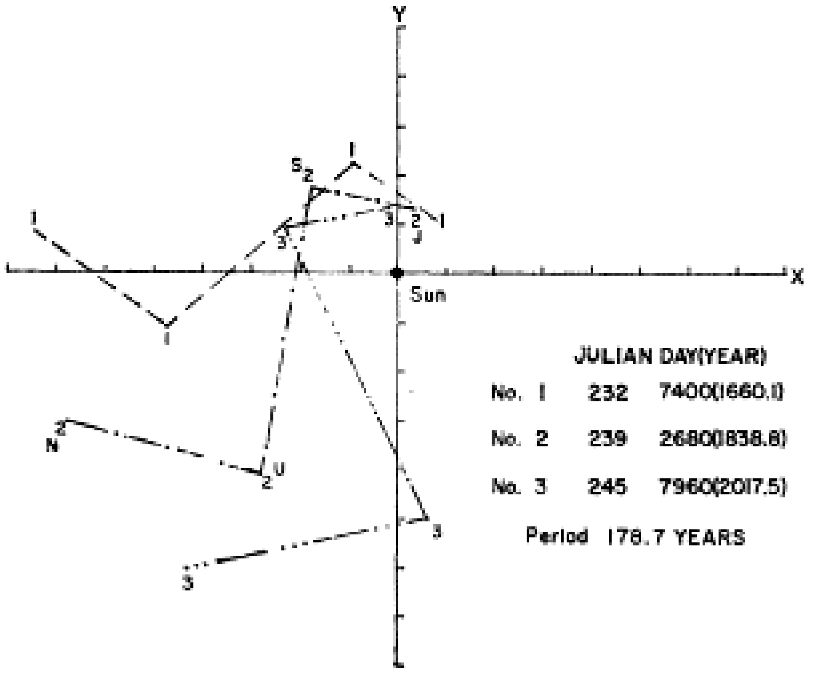

FIG. 3. Configurations of the planets Jupiter, Saturn, Uranus, and Neptune for corresponding epochs in three successive periods. The XY plane is the invariable plane.

well as the times of maximum and minimum. Since 1852 records have been kept based on the system which Wolf established. Waldmeier at Zurich is currently responsible for the compilation of data from all over the world and the determination of the definitive sunspot numbers. It is not known to the writer just how valid the observations were on which Wolf assigned the times of maxima and minima prior to 1852.

Table III gives the dates of maxima and minima from 1610 to 1957 and the predicted dates to 2009. For this discussion the maximum of 1615 has been taken as the starting point and has been designated as number one, although Waldmeier (1941) began the numbering of the cycles with the maximum of 1761.

Nicholson (1957) suggested plotting the relative sunspot numbers as positive and negative numbers to emphasize the change in magnetic polarity of the sunspot pairs. This is done, presuming early polarities, in Figs. 2(a) and 2(b). The times of maxima are indicated along the abscissas of the dL/dt and dP/dt curves. The signs of the sunspot numbers are chosen to agree with the signs of the dP/dt curve. There are four time intervals when dP/dt is positive with no sunspot maxima occurring. These are around the years 1620, 1700, 1800, and 1880. This circumstance, as predicted by these curves should occur again about 1978. These dates follow anomalous extra wiggles in the dL/dt curve. With these exceptions and allowing for uncertainties in the dates of sunspot maxima prior to 1852 the sunspot curve agrees quite well with the dP/dt curve.

In Fig. 4 the sunspot data of Figs. 2(a) and 2(b) are superimposed. The times of maxima and minima between 1610 and 1963 are indicated for the three time intervals as noted at the right end of the time axis. It so happens that the interval 1610–1963 is practically twice the 178.7-yr period as determined

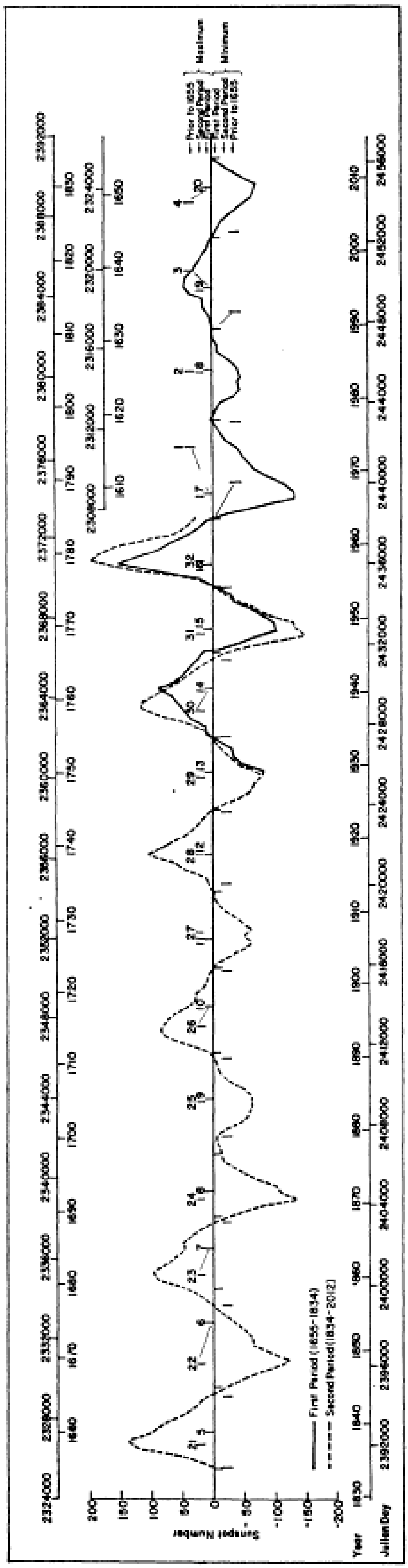

FIG. 4. The relative sunspot numbers and times of maxima and minima superimposed for the periods 1655–1833, 1833–2012 and a segment of the period prior to 1655 (1610–1655).

Table IV. Period determined from maxima and minima sunspot activity.

Function	Mean period (yr)	Standard deviation (yr)	Number of points
Minima	178.05	1.56	16
Maxima	177.28	2.28	16
Combined	177.66	1.96	32

earlier so that the times of sunspot maxima can be paired over two full periods if the sunspot curve has the same 178.7-yr period. This is found to be true. The trend of the sunspot cycles around the maxima Nos. 13, 14, 15, and 16 is very closely followed by that for the maxima Nos. 29, 30, 31, and 32. These are the only cycles for which relative sunspot numbers exist for corresponding times in the two periods.

In Table III the maxima and minima are paired so as to agree with the 178.7 period as indicated in Fig. 4. Table IV gives the average period as determined from the sunspot maxima, from the minima and from a combination of the two. The period thus found being 177.66 yr. From further consideration of Fig. 4 it is felt that Wolf may not have had sufficient data to accurately determine the dates of the minima given for 1610.8 and 1634.0 and for the maxima 1(1615.5), 3(1639.5), 6(1675.0), 7(1685.0), 10(1718.2), and 14 (1761.5). By omitting the eight period lengths which depend on these dates, there remain 24 period lengths for which the average is 178.55 yr and the standard deviation is 1.05 yr. The results may be summarized as:

The period determined from mechanical considerations,	178.77,	$\sigma=0.34$
The period determined from sunspot activity	178.55,	$\sigma=1.05$

Even if it is postulated that the sunspot activity is induced by variations in the rate of change of the sun's motion the resulting turbulence would not necessarily be expected to follow exactly the rate of change of the parameters.

The five dates around which the dP/dt curve is positive and during which intervals no sunspot maxima occur, divide the available data into groups of alternate seven and nine maxima as depicted in Table V. The groups are designated A-1, B-1, A-2 and B-2, where the A groups have seven maxima and the B groups have nine maxima. Each group begins and ends with a sunspot maximum which coincides with the negative portion of the dP/dt curve. The question arises as to the manner in which the sunspot polarities occur during the transition between groups. The polarities around the maxima Nos. 33 and 34 should prove interesting to observe. Will the cycles Nos. 33 and 34 have the same polarity pattern or will it reverse? The writer anticipates that the cycle No. 33 will reverse polarity from cycle No. 32 and that cycle No. 34 will have the same polarity pattern as cycle No. 33. Since the observations of sunspot polarity only began in 1908 near the time of the maxima No. 27, there has been no opportunity to observe the critical time in the transitions between the assigned groups in Table V.

Table V. The sunspot maxima arranged in groups.

A-1		B-1		A-2		B-2	
Maxima No.	dP/dt	Maxima No.	dP/dt	Maxima No.	dP/dt	Maxima No.	dP/dt
2	−	9	−	18	−	25	−
3	+	10	+	19	+	26	+
4	−	11	−	20	−	27	−
5	+	12	+	21	+	28	+
6	−?	13	−	22	−	29	−
7	+?	14	+	23	+?	30	+
8	−	15	−	24	−	31	−
		16	+			32	+
		17	−			(33	−)

Twenty observations of the period between maximum for the dP/dt curve [Figs. 2(a) and 2(b)] have a mean of 19.82 yr and a standard deviation σ of 2.30 yr. This is consistent with the 19.85 yr for the synodic period of Jupiter and Saturn. If the sunspot maxima from 1626.0 to 1957.5 are grouped in pairs to conform with the magnetic cycle the mean length and standard deviation of the observed periods are found to be 21.98 and 2.93 yr, respectively. The four pairs dependent on the maxima which occurred in 1675.0 and 1718.2 were omitted in order to be consistent with the computation of the 178.55-yr period for the sunspot activity as noted above. The difference in the periods of the maxima of the dP/dt curve and the magnetic cycle is attributed to the two maxima of the dP/dt curve in each of the 178-yr periods for which there are no associated sunspot maxima. Based upon the Snedecar F distribution, the standard error of 1.05 yr associated with the 178.55 yr period is significantly smaller statistically than the 2.93 yr associated with the 21.98-yr period. Thus, there are undoubtedly periodicities of low frequency in the sunspot cycle not accounted for in the shorter period of 21.98 yr that are represented in the longer 178.55-yr period. Consequently the 178.55-yr period appears to be a more realistic period for the sunspot cycle.

V. CONCLUSIONS

The relationships set forth here imply that certain dynamic forces exerted on the sun by the motions of the planets are the cause of the sunspot activity. This is supported by the 178+ yr periods in the sun's cycles.

Based on the above reasoning it may be that the behavior of the so-called irregular variable stars can be explained by assuming that they have massive satellites which are closer to the primary and consequently have shorter periods of revolution than the massive satellites in the solar system. The result would be greater agitation in the atmosphere of the star due to the variations of dynamic forces. Such forces could produce the light variations observed.

Similar preliminary studies for the earth and moon indicate that weather conditions may be dependent on such forces. These conditions are not the same at different localities since they are influenced by the distribution of land and water masses.

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